AVIRAL CLASSES
IIT-JEE | NEET | FOUNDATIONS

MATHEMATICS
CRASH COURSE

LECTURE-02

TOPICS : Matrices and Determinants

1. If $f(x)=\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is constant.

   Then $\frac{d^3}{dx^3}[f(x)]$ at $x=0$ is

   (a) p (b) $p + p^2$
   (c) $p + p^3$ (d) independent of p

2. The equations $2x - 3y + 6z = 4$, $5x + 7y - 14z = 1$, $3x + 2y - 4z = 0$, have
   (a) unique solution
   (b) no solution
   (c) infinitely many solution
   (d) none of these

3. For the equation $x + 2y + 3z = 1$, $2x + y + 3z = 2$, and $5x + 5y + 9z = 4$
   (a) there is only one solution
   (b) there exist infinitely many solutions
   (c) there is no solution
   (d) none of these

4. the system of linear equation :
   $x + y + z = 0$, $2x + y - z = 0$, $3x + 2y = 0$ has
   (a) no solution
   (b) a unique solution
   (c) an infinitely many solutions
   (d) none of these

5. If $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ then $A^2 - 5A + 61 =$

   (a) $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & -5 \\ -1 & -1 & -4 \\ -3 & -10 & 4 \end{bmatrix}$

   (c) 0 (d) 1

6. The matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$ is
   (a) nilpotent of order 3
   (b) involutary
   (c) orthogonal
   (d) idempotent

7. The matrix $A=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is

   (a) idempotent (b) involutary
   (c) orthogonal (d) nilpotent of order 3

8. If $a > 0$, $b > 0$, $c > 0$ are respectively the p^{th}, q^{th} r^{th} terms of a G.P., then the value of the determinant

   $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is

   (a) – 1 (b) 0
   (c) 7 (d) None of these

9. If α, β, γ are the roots of $x^3 + ax^2 + b = 0$, then the value of

   $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

   (a) a^3 (b) $a^2 - 3b$
   (c) $a^3 - 3b$ (d) None of these

10. If A, B and C are the angles of a triangle and

    $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0,$

    then the triangle ABC is
    (a) right anlged isosceles
    (b) isosceles
    (c) equilateral
    (d) none of thes

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TOPICS : Matrices and Determinants SOLUTION

1	D
2	--
3	D
4	C
5	A
6	A
7	A
8	C
9	A
10	B

1.

(d) $\frac{d}{dx}f(x)=\begin{vmatrix}3x^2 & \cos x & -\sin x\\ 6 & -1 & 0\\ p & p^2 & p^3\end{vmatrix}$

$\frac{d^3}{dx^3}f(x)$ at $x=0=\begin{vmatrix}6 & -1 & 0\\ 6 & -1 & 0\\ p & p^2 & p^3\end{vmatrix}=0$

i.e., independent of p.

2. The equations are equivalent to matrix equation $AX=B$.

Where $A=\begin{bmatrix}2 & -3 & 6\\ 5 & 7 & -14\\ 3 & 2 & -4\end{bmatrix}$

$X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}4\\ 1\\ 0\end{bmatrix}$

Now, $|A|=\begin{vmatrix}2 & -3 & 6\\ 5 & 7 & -14\\ 3 & 2 & -4\end{vmatrix}=\begin{vmatrix}2 & -3 & 0\\ 5 & 7 & 0\\ 3 & 2 & 0\end{vmatrix}=0.$ (by $C_3\to C_3+2C_2$)

∴ The equations either have no solution or an infinite number of solutions. To decide about this, we proceed to find (Adj. A) B

$C_{11} = 0,\ C_{12} = -22,\ C_{13} = -11$

$C_{21} = 0,\ C_{22} = -26,\ C_{23} = -13$

$C_{31} = 0,\ C_{32} = 58,\ \ C_{33} = 29.$

$$\text{Adj. } A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -22 & -26 & 58 \\ -11 & -13 & 29 \end{bmatrix}$$

$$(\text{Adj. } A)\, B = \begin{bmatrix} 0 & 0 & 0 \\ -22 & -26 & 58 \\ -11 & -13 & 29 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -114 \\ -57 \end{bmatrix} \neq 0$$

Hence the system has no solution.

3. (d) The equations are equivalent to the matrix equations $AX = B$. ...(i)

$$\text{Where } A = \begin{bmatrix} 2 & -3 & 6 \\ 5 & 7 & -14 \\ 3 & 2 & -4 \end{bmatrix},$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ 2 & -1 & 0 \end{vmatrix}$$

(by $R_2 \to R_2 - R_1, R_3 \to R_3 - 3R_1$)

$$= 3(-1+2) = 3 \neq 0.$$

Hence there is only one solution.

4. **The system is equivalent to matrix equation.**

$AX = 0$

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 0.$

Hence the system has infinitely many solutions.

5. (a) $A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

$\therefore\ A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

6. (a) Let $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$

$\therefore\ B.B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow B^2 = 0$

$\Rightarrow$ B is a nilpotent matrix of order 3.

7. (a) Let $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$\therefore \quad A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$A^2 = A$

$\Rightarrow$ A is an Idempotent matrix.

8. (c) Let A be the first term and R be the common ratio of the G.P. Then,

$a = AR^{p-1} \Rightarrow \log a = \log A + (p-1)\log R$

$b = AR^{q-1} \Rightarrow \log b = \log A + (q-1)\log R$

$c = AR^{r-1} \Rightarrow \log c = \log A + (r-1)\log R$

Now, $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$

$= \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$

[Applying $C_1 \to C_1 - \log A\, C_3$]

$= \log R \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$ [Applying $C_1 \to C_1 - C_2 + C_3$]

9. (a) Since α, β, γ are the roots of the given equation, therefore, $\alpha + \beta + \gamma = -a$, $\alpha\beta + \beta\gamma + \gamma\alpha = 0$ and $\alpha\beta\gamma = -b$.

Now, $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

$= -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$

$= -(\alpha + \beta + \gamma)\{(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)\}$

$= -(-a)\{a^2 - 0\} = a^3$.

(b) Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, to the given determinant, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ 1+\sin A & \sin B - \sin A & \sin C - \sin A \\ \sin A + \sin^2 A & (\sin B - \sin A) + (\sin^2 B - \sin^2 A) & (\sin C - \sin A) + (\sin^2 C - \sin^2 A) \end{vmatrix} = 0$$

10.

$\Rightarrow$ $(\sin B - \sin A)\,(\sin C - \sin A)$

$$\begin{vmatrix} 1 & 1 \\ 1+\sin B+\sin A & 1+\sin C+\sin A \end{vmatrix} = 0$$

$\Rightarrow (\sin B - \sin A)\,(\sin C - \sin A)\,(\sin C - \sin B) = 0$

$\Rightarrow \sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$

$\Rightarrow A = B$ or $B = C$ or $C = A$

$\Rightarrow \Delta\, ABC$ is isosceles